Rational Equations

1. $3 = \dfrac{12}{4x+5}$

$3 \cdot (4x+5) = \dfrac{12}{(4x+5)} \cdot (4x+5)$

$12x + 15 = 12$

$12x = -3$

$x = -\dfrac{1}{4}$

$SS = \left\{-\dfrac{1}{4}\right\}$

2. $\dfrac{9}{4x+2} = \dfrac{1}{2}$

$(4x+2)(2) \cdot \dfrac{9}{4x+2} = (4x+2)(2) \cdot \dfrac{1}{2}$

$(2) \cdot 9 = (4x+2) \cdot 1$

$18 = 4x + 2$

$16 = 4x$

$4 = x$

$SS = \{4\}$

3. $\dfrac{4}{9+x} = -\dfrac{1}{3x}$

$(9+x)(3x) \cdot \dfrac{4}{(9+x)} = -(9+x)(3x) \cdot \dfrac{1}{3x}$

$(3x) \cdot 4 = -(9+x) \cdot 1$

$12x = -9 - x$

$13x = -9$

$x = -\dfrac{9}{13}$

$SS = \left\{-\dfrac{9}{13}\right\}$

4. $\dfrac{y-5}{9} = \dfrac{y-7}{5}$

$9 \cdot 5 \cdot \dfrac{(y-5)}{9} = 9 \cdot 5 \cdot \dfrac{(y-7)}{5}$

$5 \cdot (y-5) = 9 \cdot (y-7)$

$5y - 25 = 9y - 63$

$-4y = -38$

$y = \dfrac{-38}{-4} = \dfrac{19}{2}$

$SS = \left\{\dfrac{19}{2}\right\}$

5. $\dfrac{4}{x-3} = \dfrac{7}{x+2}$

$(x+2)(x-3)\dfrac{4}{(x-3)} = (x+2)(x-3)\dfrac{7}{(x+2)}$

$(x+2) \cdot 4 = (x-3) \cdot 7$

$4x + 8 = 7x - 21$

$-3x = -29$

$x = \dfrac{-29}{-3} = \dfrac{29}{3}$

$SS = \left\{\dfrac{29}{3}\right\}$

6. $\dfrac{9}{2x-3} = \dfrac{6}{3x-7}$

$(2x-3)(3x-7)\dfrac{9}{(2x-3)} = (2x-3)(3x-7)\dfrac{6}{(3x-7)}$

$(3x-7) \cdot 9 = (2x-3) \cdot 6$

$27x - 63 = 12x - 18$

$15x = 45$

$x = 3$

$SS = \{3\}$

7. $\dfrac{2x}{6x^2-5} = \dfrac{1}{3x+10}$

$(6x^2-5)(3x+10)\dfrac{2x}{6x^2-5} = (6x^2-5)(3x+10)\dfrac{1}{3x+10}$

$(3x+10) \cdot 2x = (6x^2-5) \cdot 1$

$6x^2 + 20x = 6x^2 - 5$

$20x = -5$

$x = \dfrac{-5}{20} = -\dfrac{1}{4}$

$SS = \left\{-\dfrac{1}{4}\right\}$

8. $4 + \dfrac{x+2}{2} - \dfrac{x+4}{6} = 0$

$2 \cdot 6 \cdot 4 + 2 \cdot 6 \cdot \dfrac{(x+2)}{2} - 2 \cdot 6 \cdot \dfrac{(x+4)}{6} = 2 \cdot 6 \cdot 0$

$2 \cdot 6 \cdot 4 + 6(x+2) - 2(x+4) = 0$

$48 + 6x + 12 - 2x - 8 = 0$

$4x + 52 = 0$

$4x = -52$

$x = \dfrac{-52}{4} = -13, SS = \{-13\}$

9. $\frac{x+1}{2}=\frac{5x-2}{3}-\frac{3x+1}{6}$

$2\cdot3\cdot6\cdot\frac{(x+1)}{2}=2\cdot3\cdot6\cdot\frac{(5x-2)}{3}-2\cdot3\cdot6\cdot\frac{(3x+1)}{6}$

$3\cdot6\cdot(x+1)=2\cdot6\cdot(5x-2)-2\cdot3\cdot(3x+1)$

$18x+18=60x-24-18x-6$

$-24x=-48$

$x=\frac{-48}{-24}=2$

$SS=\{2\}$

10. $\frac{8}{x}+\frac{x+6}{3x}+\frac{x-4}{6x}=\frac{8}{9}$

$3\cdot6\cdot9\cdot x\cdot\frac{8}{x}+3\cdot6\cdot9\cdot x\cdot\frac{(x+6)}{3x}+3\cdot6\cdot9\cdot x\cdot\frac{(x-4)}{6x}=3\cdot6\cdot9\cdot x\cdot\frac{8}{9}$

$3\cdot6\cdot9\cdot8+6\cdot9\cdot(x+6)+3\cdot9\cdot(x-4)=3\cdot6\cdot x\cdot8$

$1296+54x+324+27x-108=144x$

$-63x=-1512$

$x=\frac{-1512}{-63}=24$

$SS=\{24\}$

11. $\frac{3}{x-1}=2-\frac{2x-5}{x+1}$

$(x-1)(x+1)\frac{3}{(x-1)}=(x-1)(x+1)\cdot2-(x-1)(x+1)\frac{(2x-5)}{(x+1)}$

$(x+1)\cdot3=(x-1)(x+1)\cdot2-(x-1)(2x-5)$

$3x+3=2x^2-2-2x^2+7x-5$

$-4x=-10$

$x=\frac{-10}{-4}=\frac{5}{2}$

$SS=\left\{\frac{5}{2}\right\}$

12. $\frac{2}{x-1}-\frac{3}{x+4}+\frac{1}{x+5}=0$

$(x-1)(x+4)(x+5)\frac{2}{(x-1)}-(x-1)(x+4)(x+5)\frac{3}{(x+4)}+(x-1)(x+4)(x+5)\frac{1}{(x+5)}=0$

$(x+4)(x+5)\cdot2-(x-1)(x+5)\cdot3+(x-1)(x+4)\cdot1=0$

$(x^2+9x+20)\cdot2-(x^2+4x-5)\cdot3+(x^2+3x-4)\cdot1=0$

$2x^2+18x+40-3x^2-12x+15+x^2+3x-4=0$

$9x=-51$

$x=\frac{-51}{9}=-\frac{17}{3}$

$SS=\left\{-\frac{17}{3}\right\}$

13. $\frac{18}{x^2-9}+1=\frac{x}{x+3}$

$(x-3)(x+3).....(x+3)$

$(x-3)(x+3)\frac{18}{(x-3)(x+3)}+(x-3)(x+3)\cdot1=(x-3)(x+3)\frac{x}{(x+3)}$

$18+(x-3)(x+3)\cdot1=(x-3)\cdot x$

$18+x^2-9=x^2-3x$

$3x=-9$

$x=\frac{-9}{3}=-3$

$SS=\{-3\}$

14. $\frac{x+1}{x^2-4}=\frac{4}{x+2}-\frac{3}{2-x}$

$(x-2)(x+2).....(x+2).....(-1)(x-2)$

$(-1)(x-2)(x+2)\frac{(x+1)}{(x-2)(x+2)}=(-1)(x-2)(x+2)\frac{4}{(x+2)}-(-1)(x-2)(x+2)\frac{3}{(-1)(x-2)}$

$(-1)(x+1)=(-1)(x-2)\cdot4-(x+2)\cdot3$

$-x-1=-4x+8-3x-6$

$6x=3$

$x=\frac{3}{6}=\frac{1}{2}$

$SS=\left\{\frac{1}{2}\right\}$

$$\frac{10}{x-3}+\frac{5}{x+1}=\frac{25}{x^2-2x-3}$$

$(x-3)...(x+1)...(x-3)(x+1)$

$$(x-3)(x+1)\left(\frac{10}{x-3}\right)+(x-3)(x+1)\left(\frac{5}{x+1}\right)=(x-3)(x+1)\left(\frac{25}{(x-3)(x+1)}\right)$$

15. $(x+1)(10)+(x-3)(5)=25$

$10x+10+5x-15=25$

$15x=30$

$x=\frac{30}{15}=2$

$SS=\{2\}$

$$\frac{3}{x^2+2x-15}+\frac{4}{x^2-9}=\frac{8}{x^2+8x+15}$$

$(x+5)(x-3)...(x+3)(x-3).....(x+5)(x+3)$

$$(x+3)(x-3)(x+5)\left(\frac{3}{(x+5)(x-3)}\right)+(x+3)(x-3)(x+5)\left(\frac{4}{(x+3)(x-3)}\right)=(x+3)(x-3)(x+5)\left(\frac{8}{(x+5)(x+3)}\right)$$

16. $(x+3)(3)+(x+5)(4)=(x-3)(8)$

$3x+9+4x+20=8x-24$

$-x=-53$

$x=53$

$SS=\{53\}$

$$\frac{4x+4}{x^2+3x+2}=\frac{x}{x+2}$$

$(x+2)(x+1)....(x+2)$

$$(x+2)(x+1)\left(\frac{4x+4}{(x+2)(x+1)}\right)=(x+2)(x+1)\left(\frac{x}{x+2}\right)$$

17. $4x+4=(x+1)(x)$

$4x+4=x^2+x$

$-x^2+3x+4=0\Rightarrow x^2-3x-4=0\Rightarrow(x-4)(x+1)=0$

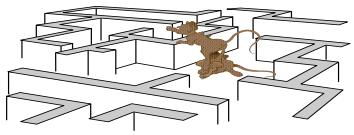
$x=4,-1$

$SS=\{4\}$

18. $\dfrac{2}{x^2-2x}-\dfrac{1}{3}=\dfrac{1}{x}$

$x(x-2)...3...x$

$3x(x-2)\left(\dfrac{2}{x(x-2)}\right)-3x(x-2)\left(\dfrac{1}{3}\right)=3x(x-2)\left(\dfrac{1}{x}\right)$



$3(2)-x(x-2)=3(x-2)$

$6-x^2+2x=3x-6$

$-x^2-x+12=0 \Rightarrow x^2+x-12=0 \Rightarrow (x+4)(x-3)=0$

$x=-4,3$

$SS=\{-4,3\}$

19. $\dfrac{3x-5}{x^2+4x+3}+\dfrac{2x+2}{x+3}=\dfrac{x-3}{x+1}$

$(x+3)(x+1)...(x+3)...(x+1)$

$(x+3)(x+1)\left(\dfrac{(3x-5)}{(x+3)(x+1)}\right)+(x+3)(x+1)\left(\dfrac{(2x+2)}{(x+3)}\right)=(x+3)(x+1)\left(\dfrac{(x-3)}{(x+1)}\right)$

$(3x-5)+(x+1)(2x+2)=(x+3)(x-3)$

$3x-5+2x^2+4x+2=x^2-9$

$x^2+7x+6=0 \Rightarrow (x+6)(x+1)=0$

$x=-6,-1$

$SS=\{-6,-1\}$

20. $\dfrac{1}{x^2+2x-8}+\dfrac{3x}{2x^2+19x+44}=\dfrac{2x}{2x^2+7x-22}$

$(x+4)(x-2)...(2x+11)(x+4)...(2x+11)(x-2)$

$(x-2)(x+4)(2x+11)\left(\dfrac{1}{(x+4)(x-2)}\right)+(x-2)(x+4)(2x+11)\left(\dfrac{3x}{(2x+11)(x+4)}\right)=(x-2)(x+4)(2x+11)\left(\dfrac{2x}{(2x+11)(x-2)}\right)$

$(2x+11)(1)+(x-2)(3x)=(x+4)(2x)$

$2x+11+3x^2-6x=2x^2+8x$

$x^2-12x+11=0 \Rightarrow (x-11)(x-1)=0$

$x=11,1$

$SS=\{1,11\}$